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Probability and Nonlocality in Many Minds Interpretations of Quantum Mechanics

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ABSTRACT

We argue that a certain type of many minds (and many worlds) interpretations of quantum mechanics, e. g. Lockwood ([1996a]), Deutsch ([1985]) do not provide a coherent interpretation of the quantum mechanical probabilistic algorithm. By contrast, in Albert and Loewer's ([1988]) version of the many minds interpretation there is a coherent interpretation of the quantum mechanical probabilities. We consider Albert and Loewer's probability interpretation in the context of Bell-type and GHZ-type states and argue that it implies a certain (weak) form of nonlocality.

1 Introduction

In this paper we shall consider two questions in the context of many minds interpretations of quantum mechanics. The first question is whether and how the notion of probability makes sense in these interpretations. We shall mainly refer to two versions of many minds interpretations: Albert and Loewer's ([1988]) stochastic version in which the minds don't supervene on physical states, and Lockwood's ([1996a,b]) version in which there is full supervenience. These two versions have been discussed in some detail in a special symposium hosted by this Journal ([1996], Vol. 47, pp. 159-248). Lockwood's approach to probabilities seems to be accepted amongst many authors in the many worlds tradition (though with different styles) e. g. Deutsch ([1985]), Zurek ([1993]), Saunders ([1998]), Papineau ([1996]), Vaidman ([1998]), and others. Our second question concerns the implications of the notion of probability in Albert and Loewer's theory these approaches on the question of nonlocality. In particular the questions is: what are the implications of Bell's theorem and the GHZ (Greenberger, Horne and Zeilinger [1989]) set up in Albert and Loewer's stochastic theory?

The structure of the paper is as follows. We first briefly present Albert and Loewer's version of the many minds interpretation and we set up the problem of interpreting the probabilities in a many minds (worlds) picture (section 2). In Section 3 we present and discuss the supervenience versions (focusing on Lockwood) of the many minds interpretation, and we argue that in these versions the probability interpretation is wanting. Then in section 4 we argue that the Albert-Loewer many minds interpretation implies a certain weak form of nonlocal correlations between subsets of minds. Finally, in Section 5 we demonstrate the nonlocality of the Albert-Loewer interpretation using the Greenberger, Horne and Zeilinger (GHZ) ([1989]) set up.

To get a quick grip on many minds interpretations consider the scheme of a generic (impulsive) measurement of the z-spin variable of an electron in noncollapsing quantum mechanics. Take a composite of quantum system, apparatus and observer (respectively) $S + M + O$ initially in the state

$$|\Psi_0\rangle = \Big(\alpha|-_z\rangle + \beta|+_z\rangle\Big) \otimes |\psi_0\rangle \otimes |\Phi_0\rangle, \tag{1}$$

where $|\alpha|^2 + |\beta|^2 = 1$. Here the $|\pm_z\rangle$ are the z-spin eigenstates, $|\psi_0\rangle$ is the ready state of M and $|\Phi_0\rangle$ is some suitable state of O's brain initiating conscious mental states. We assume that the evolution of the global state is described by the Schrödinger equation alone, i. e. there is no collapse of the quantum state. The measurement interaction between S and M takes the state (1) to the superposition

$$|\Psi_1\rangle = \Big(\alpha|+_z\rangle \otimes |\psi_+\rangle + \beta|-_z\rangle \otimes |\psi_-\rangle\Big) \otimes |\Phi_0\rangle, \tag{2}$$

where as can be seen a one-to-one correlation is brought about between the spin states $|\pm_z\rangle$ and the pointer states $|\psi_\pm\rangle$, but in such a way that the quantum states of both S and M become entangled in (2). The interaction between M and the observer O takes the global state to the final superposition

$$|\Psi_f\rangle = \alpha|+_z\rangle \otimes |\psi_+\rangle \otimes |\Phi_+\rangle + \beta|-_z\rangle \otimes |\psi_-\rangle \otimes |\Phi_-\rangle\Big), \tag{3}$$

where the $|\Phi_\pm\rangle$ are the observer's *brain* states corresponding to her *mental states*.[1] As can be seen the state (3) is now also entangled and the reduced state of the observer is truly mixed. If one takes this theory to be complete *simpliciter* (called by Albert ([1992]) the bare theory), then one faces the measurement problem since the measurement has no definite result. On this view the quantum statistical algorithm which is an algorithm about the probabilities of measurement results makes no sense (see Albert (*ibid*, Chapter 6) for more details). Thus in order to avoid the measurement problem, one needs somehow to supplement the bare theory's description.

2 Albert and Loewer's Interpretation

Many minds interpretations take the bare theory to be indeed complete and exactly true but only with respect to the *physics,* including the physics of the brain. In other words, the quantum state never collapses and no hidden

[1] We shall assume throughout that the set of brain states corresponding to all possible outcomes of all possible experiments forms a basis in the brain's Hilbert space. This is a preferred basis in the Hilbert space corresponding to the (subjective) mental states associated with conscious perception.

variables are added to the quantum mechanical description. To supplement the bare theory, further assumptions are made with respect to the relation between the brain states $|\Phi\pm\rangle$ and O's mental states. Albert and Loewer [1988] make the following two assumptions:

AL1 The brain states $|\Phi_{\pm}\rangle$ corresponding to O's mental states are associated at all times with a continuous *infinity* of nonphysical entities called *minds* (even for a single observer).

AL2 Minds do not obey the Schrödinger evolution (in particular, the superposition principle) but evolve in time in a genuinely probabilistic fashion. For a given measurement, involving a conscious observer, there is one specific probability measure, given by the Born rule, that prescribes the chances for each mind to evolve from an initial $|\Phi_0\rangle$ to a final brain state $|\Phi_i\rangle$.

In the measurement scheme above each single mind corresponds initially to the state $|\Phi_0\rangle$ and evolves in a *stochastic* fashion to one of the two final brain-mental states $|\Phi\pm\rangle$ with the usual Born probabilities: $|\alpha|^2$ for a $+$ result and $|\beta|^2$ for a $-$ result. The divergence of the minds occurs during the evolution of the global state from (2) to (3). Let us denote by $|\Phi(m)\rangle$ a quantum brain state indexed by a subset m of the set of minds. The complete description of the post-measurement state includes the quantum state, and the corresponding subsets of the set of minds. Therefore, one needs to replace (3) with

$$|\Psi_f(m,n)\rangle = \alpha|+_z\rangle \otimes |\psi_+\rangle \otimes |\Phi_+(m)\rangle + \beta|-_z\rangle \otimes |\psi_-\rangle \otimes |\Phi_-(n)\rangle. \qquad (4)$$

Here we use the notation $\Psi_f(m,n)$, to make explicit the Albert-Loewer idea that the quantum brain states correspond to, and are indexed by, subsets of the set of minds. In the state (4) we see that the minds in the subset m follow the brain state in the $+$ branch of the superposition, and those in the subset n follow the brain state in the $-$ branch. The evolution of the minds is genuinely stochastic. This we take to mean that before the minds actually diverge into the branches in the state (2), there is no determinate fact of the matter about which branch each one of the minds will eventually follow. The membership of a given mind in the subset m (or n) becomes a fact at the same time that the final state (4) obtains. The standard quantum mechanical probability is thus understood as the chance for each *single* mind to end up in either the m-subset or the n-subset in the state (4).

The motivation for assuming a multiplicity of minds here, rather than a single mind that literally chooses one of the branches in the state (4), is this. First, in a single mind theory all the brain states of the observer after a split are mindless, except for the one that is actually tracked by the mind. This leads to the so-called *mindless hulk* problem (Albert [1992], p. 130). Suppose, for example, that a second observer also measures the z-component of spin when the state (4) obtains, and take $\alpha = \beta = 1/\sqrt{2}$. Then the dynamical equations of motion guarantee that the brain states of the two observers will be *correlated* with certainty. But there is probability one-half that their minds will not track the same branch of the state (see *ibid*, p. 130). Second, Bell's theorem implies that in a single mind theory the *correlations* between the minds of two observers on the two wings of a Bell-type experiment will satisfy the quantum predictions only by allowing strong nonlocal dependence between the trajectories of the minds (of the kind exhibited in usual hidden variable theories; see Lockwood ([1996a])). The Albert and Loewer theory avoids such strong nonlocality by associating with each brain state of an observer after a split an infinity of minds, and by postulating a genuine stochastic dynamics for the minds (see section 4 for more details on this particular issue).

To sum up, we can characterize Albert and Loewer's interpretation as follows. (i) There are no collapses, but the expansion of the global state (4) in terms of the brain states $|\Phi_i\rangle$ and their *relative* states, e. g. the pointer states $|\psi_i\rangle$, is taken to describe our experience. (ii) There is a random element built into the theory. The fact that the time evolution of the minds is stochastic is depicted by the quantum mechanical probabilities. (iii) The probability measure is *conditional* on a given measurement. (iv) Individual minds (unlike the *proportion* of minds) do not supervene on brain states. This means that an m-mind can be exchanged with an n-mind in the superposition (4) with no corresponding change in the physics. In fact, the chance interpretation of the probability measure in AL2 *implies* this failure of supervenience in Albert and Loewer's version. This is the so-called *dualistic* aspect of this version (Lockwood [1996a], Loewer [1996]).

Let us see how the Albert-Loewer approach bears on the relationship between branching and relative frequencies. As is well known, this is a major problem in the Everett picture where the number of branches resulting from a given quantum measurement is not related to the quantum mechanical probabilities. For example, in a measurement with two possible results the quantum state will consist of two branches corresponding to the two results of the measurement irrespective of the probabilities for each result. This has the consequence that in a repeated measurement the relative frequencies of an outcome (along a branch) will most likely mismatch the quantum mechanical predictions. It then follows that the empirical success of quantum mechanics, as observed by us, must be viewed as a miracle since most of the Everett branches will not exhibit the right quantum mechanical frequencies.

This problem can be solved if one postulates that the standard Born rule, applied for each measurement, represents the probability of the branch. In other words, one simply brings in the probability as an extra postulate in addition to the branching. For example, consider a process where at t_1 a measurement with two possible results and probabilities $\frac{1}{3}$ and $\frac{2}{3}$ is performed. Then follows another measurement at t_2, with three possible results with identical probabilities $\frac{1}{3}$, and so on. A suitable law of large numbers can be proved for such a tree. In particular, in a sequence of *identical* measurements, the frequency on almost all branches will be close in value to the quantum mechanical probability distribution on the set of measurement outcomes. (e. g. Everett ([1957]), DeWitt ([1970]), Hartle ([1968])). The Albert-Loewer approach provides a simple explanation: Each individual mind performs a (classical) random walk on the tree, with the probabilities indicated on the branches. The fact that a typical mind perceives the quantum mechanical frequencies simply follows from the theory of random walks (or branching processes).

However, *quantum mechanics assigns, in advance, probabilities to all possible measurement trees*. Given a quantum state of the system, we can calculate in advance the probabilities of all possible sequences of measurements. For example, insted of the measurement just considered, we can perform at time t_1 a measurement with three possible outcomes whose probabilities are $\frac{1}{5}, \frac{3}{5}, \frac{1}{5}$. At t_2 we do not measure anything, and then at t_3 we perform a particular measurement with two possible outcomes, and so forth. The probability for each step in the sequence is known at the outset. Schrödinger ([1935]) noticed that "at no moment in time is there a collective distribution of classical states which would be in agreement with the sum total of quantum mechanical predictions". This means that while each measurement sequence can be seen as a classical random walk, there is no (non contextual) classical probability distribution which assigns the correct probabilities to all the branches of all possible trees simultaneously. This is a major difference between the quantum concept of probability and the classical one. One manifestation of this difference is the violation of Bell inequalities. (Pitowsky [1994]).

We can see why the assumptions of Albert and Loewer are almost inevitable. Suppose, contrary to AL2, that the trajectory of each mind is predetermined before measurement. Now, consider Alice and Bob who participate in a typical EPR experiment, Alice on the left and Bob on the right. In each run they can each choose a direction along which to measure the spin. We now face the task of choosing, *in advance*, the appropriate subsets for each person's set of minds, corresponding to each possible result, in each possible choice of directions. From Bell's theorem it follows that the only way to do that, and obtain the right probabilities, is to violate Bell's inequality. In the present context this means that the trajectories of some Alice minds depend on Bob's choice of direction and vice versa. What we have, in other words, is a non local hidden variable theory in disguise (with the minds playing the role of hidden variables). Note that this formal argument does not depend on any spatial characteristic of the minds themselves, or lack thereof. This is part of the reason why Albert and Loewer assume that the membership of a given mind in a given subset becomes a fact only at the same time that the final state (e.g. state (4)) obtains. That is, the random partition of the set of minds into the appropriate subsets is conditional on which observable is actually measured. They do not assume that there is a distribution on the set of minds that explains the sum total of the quantum mechanical predictions.

3 Probabilities in Lockwood's Version

We now turn to analyzing the approach to probabilities in all version of the many minds interpretation which assume complete supervenience of the mental on the physical.[2] For convenience we shall focus on Lockwood's ([1996a,b]) quite explicit approach to probabilities, but our analysis can be applied *mutatis mutandis* to other versions. Lockwood aims explicitly at a picture in which there is full supervenience of the minds on the brain states (or the corresponding branches) and there is absolutely no stochastic behaviour of the minds. Such versions assume:

SUP1 The brain states $|\Phi_\pm\rangle$ corresponding to mental states are associated at all times with a continuous infinity of nonphysical entities called minds. The minds completely *supervene* on the brain states $|\Phi_\pm\rangle$.

Lockwood's idea of supervenience of the minds on the $|\Phi_\pm\rangle$ means that he rejects the description of Albert and Loewer given by the state (4) and adopts instead the description given by (3) as a *complete* description of the physics *and* of the minds. In other words, in Albert and Loewer's theory brain states are indexed by subsets of minds, whereas in Lockwood's theory subsets of minds are indexed by brain states.

To make sense of the quantum mechanical probabilities Lockwood defines a probability measure over subsets of the minds as follows.

[2]These versions include possibly the early formulations by Everett ([1957]) and DeWitt ([1970]), as well as later versions by Zeh ([1973]), Deutsch ([1985]), Zurek ([1993]), Donald ([1995], [1999]) Vaidman ([1998]), Saunders ([1998]), and also post-Everett approaches of consistent (decoherent) histories, e. g. by Griffiths ([1984]), Gell-Mann and Hartle ([1990]).

SUP2 The standard Born rule defines a unique probability measure over subsets of minds, such that for any measurement (involving a conscious observer) the measure prescribes the *proportions* of minds following each final branch of the superposition. For example, in the post-measurement state (3), the subset of minds following the + branch is assigned the measure $|\alpha|^2$ and the subset of minds following the $-$ branch the measure $|\beta|^2$.

Thus, for Lockwood too, the quantum measure describes how many minds follow each branch of the post-measurement state (such as state (3)). Let us also note that Deutsch's ([1985]) probability measure, as well as many other supporters of the supervenience view, is essentially the same, except that it is sometimes defined over subsets of so-called *worlds*.

Lockwood's version differs from Albert and Loewer's with respect to the interpretation of probability in that the evolution of the minds is *not* genuinely random in the Albert-Loewer sense. However, in versions that assume supervenience it is not at all clear what the dynamics of the minds is. Nor is it clear how one could make the dynamics compatible with supervenience. We consider below two possible interpretations.

1) Let Λ be the set of minds of the observer. Minds supervene on brain states. This means that with each brain state $|\Phi\rangle$, which corresponds to a conscious perception of a measurement outcome, Lockwood associates a subset $m(|\Phi\rangle) \subset \Lambda$. Suppose that $|\Phi_+\rangle$ is the brain state of an observer perceiving 'spin up' (for a quantum system and apparatus in a given state, on a given day, in a given weather, and so on (including whatever it takes to specify the brain state uniquely). Then the probability measure of the subset $m(|\Phi_+\rangle)$ is $|\alpha|^2$. Now, if we assume that the association $|\Phi\rangle \to m(|\Phi\rangle)$ of brain states with subsets of minds is fixed in advance of any measurement, as suggested by supervenience (see **SUP1**), we run into a problem. We require, in fact, a choice of a probability measure on the set of minds that will be in agreement with the totality of quantum mechanical predictions. This implies non locality and contextuality in the dynamics of the minds in

the way that was explained at the end of the previous section.[3]

2) So perhaps one could endorse what Loewer ([1996]) calls the *instantaneous minds* view. (As far as we know Lockwood doesn't subscribe to this view.) On this view it is assumed that minds do not persist in time in the sense that there is no unique succession relation between any one of the minds at an earlier time and any one of the minds at a later time. One could think about this view as denying that a label can be attached to each of the minds at an early time that would distinguish it from all but one of the minds at a later time. As a result this view holds that there are no facts of the matter concerning the evolution of a single mind between any two times, and in particular between times during which the quantum state evolves into superpositions of branches indexed by different mental states.

Consider for instance the time evolution taking the state (2) at time t_1 to the final state (3) at time t_2. On the instantaneous view all we can say about the behaviour of the minds is that at t_1 the total set of the minds is associated with the state $|\Phi_0\rangle$, and at t_2 a subset of the minds with measure $|\alpha|^2$ is associated with the final brain state $|\Phi_+\rangle$ and a subset with measure $|\beta|^2$ with the brain state $|\Phi_-\rangle$. However, there is no fact of the matter as to which mind in (2) evolved into which branch in (3) in the sense that there is no real mapping of the minds at t_1 to any one of the two subsets in t_2 (see e. g. Lockwood ([1996a], p. 183), but compare Lockwood ([1996b, pp. 458-9])).

It seems, however, that Lockwood would also want to maintain that Λ, the set of all minds, is itself time invariant. To put it differently, minds are not created and destroyed through time (presumably, until the person dies). Now, this is utterly inconsistent with the instantaneous minds view. The first axiom of set theory, the axiom of equality, states that two sets are equal when they have the same elements. If Λ at t_1 is identical to Λ at t_2 there always exists a mapping, or a succession relation, between the minds at the

[3]Notice that even if the minds' labels were to fix only probabilistically the evolution of the minds, Bell's theorem would still apply, as long as the postulation of a probability measure that explains all predictions is not dropped.

two times. Simply take the identity mapping! Likewise, there is always a fact of the matter regarding which element $\lambda \in \Lambda$ is an element of the subset $m(|\Phi_+\rangle)$.

And so, if Λ is time invariant, each mind in fact *is* labeled through time just as Albert and Loewer insist. But then the question of whether such minds evolve in a genuinely stochastic fashion or in a deterministic fashion is still pertinent (see Butterfield [1996] for an extended discussion of this point). In other words, on such a view one has to provide a clear answer to the question whether the proposed dynamics of the minds conforms to the Albert-Loewer stochastic type, in which case one has to give up on supervenience. Alternatively, one could adopt the deterministic version, in which case the evolution of the minds, in e. g. EPR situations, is sometimes determined by remote spacelike separated events.

But perhaps one would insist that Λ is not time invariant, and at each time t there exists a different set of minds Λ_t. Minds are born with every experiment, they briefly supervene on the brain state, and then die like butterflies. What precisely such a theory *explains* is not clear to us.

It is entirely possible that supporters of supervenience intend an interpretation that is completely different from those that we have discussed. Lockwood [(1996a, pp. 183-4)] seems to suggest that minds may be like bosons having no individual identity. But this, really, seems to us a conflation of the explanation with the problem. It is precisely the strangeness of entangled states of the kind exhibited by identical bosons which suggests theories like the many minds interpretation in the first place. What would we achieve if the minds lived in a Hilbert space?

Nevertheless, it might be that some non-classical interpretation of probability is compatible with the idea of supervenience in the Everett tradition (see e. g. Saunders [(1998]), Lockwood ([1996b])), and would be fruitful in the sense that it includes enough of the content of our usual, classically interpreted probabilistic assertions. However, as we have just stressed, the Albert-Loewer *stochastic* and *non-supervening* character of the minds, as stressed by Albert and Loewer, seems to be necessary, if one wishes to have probabilities in the usual sense of the word, while keeping the theory local in Bell's sense.

4 Sets of Minds and Their Correlations

We now turn to analyze more specifically the question of locality in the Albert-Loewer version. Albert and Loewer ([1988]) and Albert ([1992, p. 132]) (see also Maudlin ([1994])) argue that their version is completely local. On the other hand as we saw their chancy evolution of the minds is designed to deliver the standard quantum mechanical predictions which, we know, violate the Bell inequality. The stochastic evolution of the minds which occurs only at or after an actual choice of measurement solves the problem of Bell's nonlocality. However, in what follows we shall argue that there is a *weaker* notion of locality that is violated even in the Albert-Loewer version.

Let us start with Albert and Loewer's argument. Consider the singlet state of the two particle system:

$$\sqrt{1/2}(|+_z\rangle_1|-_z\rangle_2 - |-_z\rangle_1|+_z\rangle_2), \tag{5}$$

and suppose that observer 1 (Alice) measures some spin observable of particle 1, and observer 2 (Bob) measures some (not necessarily the same) spin observable of particle 2. The overall state after these two measurements will be a superposition of branches (in general four branches). In each branch each of the two measurements has some definite outcome. Now, Albert and Loewer argue that no matter which observable gets measured by Bob, the chances of Alice to see a $+$ result or a $-$ result are exactly one-half. On this picture this means that one-half of the minds of Alice will see an up result and one-half will see a down result *independently* of the measurement of Bob. And the same goes for Bob.[4] Moreover, Albert and Loewer stipulate that the evolution of the minds of each observer (which recall is stochastic) is controlled by the local *reduced* (physical) state of that observer *alone* (see section 2). And as we have just argued this is sufficient to ensure that the evolution of the minds will satisfy the frequencies predicted by the quantum mechanical algorithm. According to Albert and Loewer, this is as far as the many minds picture goes.

In particular, Albert and Loewer (*ibid*) (see also Albert ([1992], p. 132)) argue that there are just *no* matters of fact about the correlations of the minds of Alice and Bob (nor about the outcomes of the measurements). And if no such correlations obtain, we agree that the Albert-Loewer picture is *ipso facto* local and Bell's theorem is simply irrelevant at this stage. The correlations which Bell's theorem is about will obtain only in a *local* way between the minds of one observer and the *physical* reports of the other (see (B) below)). Once such a correlation occurs the many minds picture gives the correct quantum mechanical predictions in a completely local way.

We shall now argue, however, that there are *correlations* between subsets of the minds of Alice and Bob. It is true that the chance distribution of each of the observers' minds is independent of the measurement on the other wing. For the reduced state of each of the observers is independent of the interactions occurring on the other wing[5], and according to Albert and Loewer's dynamical rule for the minds the evolution of the minds of each observer is controlled solely by the reduced (local) state of that observer, and is independent of the state of the other observer. With each experiment performed by an observer the set of minds associated with the observer's brain splits into many subsets, one for each possible outcome, and the probability of each outcome is just the measure of the corresponding subset.

Take now an EPR setting in which Alice and Bob happen to measure the *same* observable. The overall state of the two particle system and the two

[4] And of course this fact does not depend on the time order of the two measurements (or the reference frame in which we choose to describe the measurements).

[5] This follows from the so-called no-signaling theorem in standard quantum mechanics (see, e. g. Redhead ([1987]).

observers will evolve into a superposition of only two branches:

$$\sqrt{1/2}(|+_z\rangle_1|-_z\rangle_2|\Phi_+\rangle_1|\Phi_-\rangle_2 - |-_z\rangle_1|+_z\rangle_2|\Phi_-\rangle_1|\Phi_+\rangle_2). \tag{6}$$

(A) Consider a single run of the experiment. When Alice measures spin her set of minds splits into two subsets of size one-half. Call them $A+$ and $A-$. Similarly for Bob, whose set of minds splits into $B+$ and $B-$ with the same proportions. If we follow the track of a single Alice-mind we see that it ends up in the subset $A-$ with probability one-half believing a $-$ result (and similarly for the minds in $A+, B+$ and $B-$). If the same measurements are repeated N times the minds in each of these subsets trace paths connecting N vertices on the binary tree of possible results.

(B) Consider for example Alice's minds in the subset $A-$ immediately after her measurement. These minds may develop predictions about Bob's reports concerning the result(s) of his measurement. In particular, an Alice-mind in $A-$ ($A+$) will predict with certainty that Bob will report a $+$ ($-$) result. The same goes with respect to Bob's minds. According to quantum mechanics these predictions will be confirmed with certainty. In order to explain these predictions Albert and Loewer consider what happens to the quantum state immediately after Alice and Bob communicate to each other their results. The quantum mechanical equations of motion result in a state with the same form as the state (6). In particular, in that state Alice's brain state corresponding to the $-$ ($+$) result will be perfectly correlated to Bob's *report* state corresponding to the $+$ ($-$) result. Therefore, when the final state obtains, the probability that Alice's $A-$ ($A+$) minds will actually end up perceiving Bob as reporting a $+$ ($-$) result is equal to the usual quantum mechanical probability. That is, in this case Alice's predictions of Bob's reports will be confirmed with probability one. Thus, Albert and Loewer argue, there need be no facts about correlations between the minds in $A-$ ($A+$) and the minds in $B+$ ($B-$) in order to reproduce the quantum predictions (Albert [1992], p. 132)), for it suffices that there is a correlation between minds and report-states. Note that this applies both before and after the observers communicate to each other their results.

So far this is uncontroversial. In the remainder of this paper, the weak minds correlations we shall talk about can be understood as referring to the above correlations between one observer's minds and another's reports.

(C) However, Albert and Loewer's conclusion that there are no correlations between sets of minds is not entirely in line with other aspects of their theory, namely their position on the mindless hulk problem (see section 2). In particular, it is not clear why it was considered to be a problem in the first place. The problem, recall, is that if each observer has only a single mind, then there is probability of one-half (given the state (6)) that Alice's mind will follow the branch which is *not* followed by Bob's mind.[6] For example, Alice's mind may perceive a $-$ result and so may Bob's mind. When they meet, however, Alice's mind will with certainty perceive Bob as reporting a $+$ result with certainty (this is the mindless hulk).

An analogous problem appears in Albert and Loewer's many minds theory. According to Albert and Loewer there is no fact of the matter about whether or not Bob's $+$ report, as perceived by Alice's $A-$ minds, corresponds to Bob's $+$ minds. All we know is that Bob's report is associated with some minds, but Albert and Loewer do not allow us to say that these minds are $+$ minds. Suppose there are only correlations between Alice's minds and Bob's reports (and vice versa), as Albert and Loewer say, but not between Alice's sets of minds and Bob's sets of minds. This means there will be a Bob $+$ mind who witnesses Alice reporting a $-$ result, while the latter report is associated with a $+$ mind of Alice. This is analogous to the mindless hulk problem. In the single mind case a mindless brain state is producing a definite $-$ report , and in the many minds case a brain state associated with a $+$ mind produces a $-$ report. We believe that if the first is a problem, then so is the second.

In order to solve this problem we have to assume correlations between sets of minds as given by the quantum mechanical predictions. We call these correlations *weak minds-correlations* (weak nonlocality).[7] Notice that

[6] Note that this is already a statement about correlations between the minds.

[7] Note that in the versions by, e. g. Lockwood ([1996a]), Zurek ([1993]), Vaidman ([1998]), Saunders ([1998]), Donald ([1999]), the weak minds-correlations follow immediately from supervenience.

without weak minds-correlations a single mind theory with the same *local* dynamics can reproduce the quantum predictions (and the minds-to-reports correlations) just as well.[8]

Thus in the many minds picture the (weak) minds-correlations are necessary in order to provide a genuine solution to the mindless hulk problem. These correlations allow us to say, in the EPR case above for example, that the path of an Alice mind in $A-$ $(A+)$ is the mirror image of the path of a Bob mind in $B+$ $(B-)$. It is in fact an advantage of Albert and Loewer's theory that introducing such correlations doesn't violate locality in Bell's sense. This is possible because the multiplicity of the minds, as it were, compensates for the local dynamics. Bell's theorem requires the existence of a single probability measure defined over all possible (actual and counterfactual) measurements. This is denied by Albert and Loewer's local dynamics of the minds. In this way the derivation of a Bell inequality is formally blocked.

5 Many Minds and GHZ

An even more revealing case of weak nonlocal correlations between the minds in Albert and Loewer's theory is that of Greenberger, Horne and Zeilinger (GHZ) ([1989]) (See also Mermin ([1990])). In this case we will show that the quantum mechanical correlations and algebraic relations between the observables entail nonlocal constraints on the distribution of the minds into subsets corresponding to the results of measurements. Moreover, since in Albert and Loewer's theory the individual minds retain their identity throughout time, these constraints have an implication even for *individual* minds.

[8] The argument for this is the same as the one given in (B) above in the case of the many minds theory.

Consider the GHZ-state

$$\sqrt{1/2}(|+_z\rangle_1|+_z\rangle_2|+_z\rangle_3 - |-_z\rangle_1|-_z\rangle_2|-_z\rangle_3), \tag{7}$$

where the kets $|\pm_z\rangle_i$ ($i = 1, 2, 3$) denote the z-spin state of particle i. Suppose that the three particles are located in space-like separated regions. In this state standard quantum mechanics predicts a $+$ or $-$ result of the local measurements of the z-spin of each particle with probability one-half (with collapse onto the corresponding branch). And likewise for measurements of the x- and y-spins of each particle.

However, for this specific state standard quantum mechanics also predicts certain correlations between the results of the three observers, in particular the correlations described by the algebraic relations

$$\begin{array}{lrcl} \text{(XXX)} & \sigma_x^1\sigma_x^2\sigma_x^3 & = & -1 \\ \text{(XYY)} & \sigma_x^1\sigma_y^2\sigma_y^3 & = & 1 \\ \text{(YXY)} & \sigma_y^1\sigma_x^2\sigma_y^3 & = & 1 \\ \text{(YYX)} & \sigma_y^1\sigma_y^2\sigma_x^3 & = & 1 \end{array} \tag{8}$$

It is easy to see that the state (7) is an eigenstate of the above four observables with eigenvalues indicated on the right. Assuming that the local observables σ_x^i, σ_y^i take on definite values that are fixed *locally*, and that the values satisfy the correlations (8), we can easily derive the contradiction

$$-1 = \sigma_x^1\sigma_x^2\sigma_x^3 = \sigma_x^1\sigma_y^2\sigma_y^3 = 1. \tag{9}$$

This is the GHZ simplification of Bell's theorem.

In standard quantum mechanics (with a collapse postulate) this contradiction is avoided, since the local observables σ_x^i, σ_y^i are assigned no definite individual values in the initial state (7), and the collapse itself is *nonlocal* in the following sense. The result of the (local) measurements σ_x^i or σ_y^i on each wing of the experiment depends on which observables get measured on the two other wings and on their outcomes. The correlations between the results of the measurements on the three wings satisfy exactly the correlations given by (8).

But now what happens in Albert and Loewer's many minds theory in which there is no collapse of the state? If the three observers measure spin along the XXX-directions, the uncollapsed quantum state, when these measurements are over, must be:

$$
\begin{aligned}
|\Psi\rangle &= 1/2(|+_x\rangle_1|-_x\rangle_2|+_x\rangle_3|\Phi_+\rangle_1|\Phi_-\rangle_2|\Phi_+\rangle_3 + \\
&\quad +|-_x\rangle_1|+_x\rangle_2|+_x\rangle_3|\Phi_-\rangle_1|\Phi_+\rangle_2|\Phi_+\rangle_3 + \\
&\quad +|+_x\rangle_1|+_x\rangle_2|-_x\rangle_3|\Phi_+\rangle_1|\Phi_+\rangle_2|\Phi_-\rangle_3 + \\
&\quad +|-_x\rangle_1|-_x\rangle_2|-_x\rangle_3|\Phi_-\rangle_1|\Phi_-\rangle_2|\Phi_-\rangle_3)
\end{aligned} \tag{10}
$$

(written in the x-spin bases).

In this state the marginal probabilities for a $+$ or $-$ result of the local measurements of each observer is one-half. In the Albert and Loewer theory this means that for each observer the proportions of the minds perceiving a $+$ or $-$ result are one-half. But because of the constraint XXX in (8) the state (10) consists of only four branches in each of which there is a strict anti-correlation between the sign of the result perceived by, say the subset of minds of observer 1, and the sign of the product of the results perceived by subsets of minds of observers 2 and 3. These anti-correlations are brought about since the quantum mechanical amplitudes of the branches with strict correlations in the state (10) is zero. Now as we have argued in section 4 the anti-correlations in the state (10) mean that the *subsets of minds* of the three observers that correspond to these branches are weakly correlated (in the sense of (C) in section 4). A similar analysis applies for the alternative measurements along the directions XYY, YXY and YYX, but in these latter cases quantum mechanics predicts strict correlations between the (sign of the) results perceived by subsets of minds of any given observer and the sign of the product of the results perceived by subsets of minds of the other two observers (see equation (8)). Call the four possible measurements XXX, XYY, YXY and YYX *scenarios* and number them from 1 to 4 respectively.

Consider the implications of the GHZ analysis for the Albert and Loewer theory. Given the quantum mechanical correlations described by (8), we obtain that in each scenario the set of triples of minds $M_A \times M_B \times M_C$ (where M_A is the set of Alice's minds, etc.) is partitioned into four disjoint subsets. Thus, in the first scenario XXX we can write

$$\begin{aligned} M_A \times M_B \times M_C \;=\; & M_A^+(1) \times M_B^+(1) \times M_C^-(1) \cup \\ & \cup M_A^+(1) \times M_B^-(1) \times M_C^+(1) \cup \\ & \cup M_A^-(1) \times M_B^+(1) \times M_C^+(1) \cup \\ & \cup M_A^-(1) \times M_B^-(1) \times M_C^-(1). \end{aligned} \tag{11}$$

Here, $M_A^+(1)$ stands for the subset of Alice's minds that observe $+$ in the first scenario. In the other three scenarios XYY, YXY and YYX we have

$$\begin{aligned} M_A \times M_B \times M_C \;=\; & M_A^+(j) \times M_B^-(j) \times M_C^-(j) \cup \\ & \cup M_A^-(j) \times M_B^+(j) \times M_C^-(j) \cup \\ & \cup M_A^-(j) \times M_B^-(j) \times M_C^+(j) \cup \\ & \cup M_A^+(j) \times M_B^+(j) \times M_C^+(j), \end{aligned} \tag{12}$$

where $j = 2, 3, 4$ index the scenario.

We stress that this fact, namely that the set of triples of the minds is partitioned into four disjoint subsets in each scenario, is completely *independent* of the question which scenario (if any) will be eventually performed. We may assume, for example, that this question is settled *deterministically* by the initial conditions. This assumption is perfectly consistent with Albert and Loewer's theory.

To see how the weak non-locality is manifested here consider, for example, the set

$$M_A^-(1) \times M_B^+(1) \times M_C^+(1) \cap M_A^-(2) \times M_B^+(2) \times M_C^-(2) \cap \tag{13}$$

$$\cap M_A^-(3) \times M_B^-(3) \times M_C^+(3) \cap M_A^+(4) \times M_B^+(4) \times M_C^+(4).$$

It consists of an intersection of four partition elements, one from each scenario. If a triple of minds belongs to this set then Bob's mind perceives $+$ in the x-direction in scenario 1 and $-$ in the x-direction in scenario 3, while Carol's mind perceives $-$ in the y-direction in scenario 2 and $+$ in the y-direction in scenario 3. By GHZ, this is true for any set like (13), an intersection of four partition elements, one from each scenario. For each triple in such a set at least one of the observers flips signs in the same local measurement, while changing the scenario. In other words, either Alice's mind is flipping when changing from scenario 1 to 2, or from scenario 3 to 4; or Bob's mind is flipping when changing from 1 to 3, or from 2 to 4; or Carol's mind is flipping when changing from 1 to 4, or from 2 to 3.

We can estimate the size of intersections like (13). There are $4^4 = 256$ such sets, and between them they cover all the logical possibilities. Therefore, at least one of those sets has probability $\geq \frac{1}{256}$. This type of information is available in Albert and Loewer's theory, while completely absent from quantum mechanics. In this sense their interpretation is a hidden variable theory of sorts.

We stress that the above argument cannot be circumvented by merely pointing out that the evolution of the minds in the Albert and Loewer theory is stochastic. As can be seen in the previous paragraph the fact that any given mind has probability of $1/2$ to perceive a $+$ or a $-$ result in each scenario is already taken into account in our argument. The weak nonlocal correlations between the minds is not only a statistical constraint on the *proportions* of minds. Rather, it is a constraint on the correlations between *individual minds across scenarios.*

However, since the dynamics of the minds in the Albert and Loewer theory is genuinely stochastic, it is contingent which of the individual minds of each observer flip their sign between different scenarios. In other words, *our argument does not involve counterfactual reasoning about individual minds.* Hence, our use of the term 'scenario', and not 'possible world'. The latter would appropriately correspond to a particular assignment of a value to each individual mind in a particular scenario. What changes from one scenario to another are the *sets of minds*, and each scenario corresponds, therefore, to a multitude of possible worlds. The weak sense of nonlocality obtained in this way is hidden behind the stochastic evolution of the minds, and eventually behind the stochastic results we perceive.

Finally, since the weak form of these nonlocal correlations is a feature of the *uncollapsed* quantum state, such as (10), the evolution of the minds need not exhibit, not even in principle, a dependence on a reference frame. In this sense a theory with such correlations might be written in a genuine relativistic form. But this is another issue.

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References

Albert, D. [1992]: *Quantum Mechanics and Experience*, Cambridge, Mass.: Harvard University Press.

Albert, D. and Loewer, B. [1988]: 'Interpreting the Many Worlds Interpretation', *Synthese*, **77**, pp. 195-213.

Butterfield, J. [1996]: 'Whither the Minds?', *British Journal for the Philosophy of Science*, **47**, pp. 200-21.

Deutsch, D. [1985]: 'Quantum Theory as a Universal Physical Theory', *International Journal of Theoretical Physics* **24**, pp. 1-41.

DeWitt, B. [1970]: 'Quantum Mechanics and Reality', *Physics Today*, **23**, pp. 30-5

Donald, M. J. [1995]: 'A Mathematical Characterization of the Physical Structure of Observers', *Foundations of Physics*, **25**, pp.529-71.

Donald, M. J. [1999]: 'Progress in a Many Minds Interpretation of Quantum Theory', *Los Alamos E-Print Archive*, http://www.arxiv.org/abs/quant-ph/9904001.

Everett, H., III [1957]: 'Relative State Formulation of Quantum Mechanics', *Reviews of Modern Physics*, **29**, pp. 454-62.

Gell-Mann, M. and Hartle. J. [1990]: 'Quantum Mechanics in the Light of Quantum Cosmology', in W. Zurek (*ed.*), *Complexity, Entropy and the Physics of Information, SFI Studies in the Physics of Complexity* (Vol. VIII), Redwood City, CA: Addison-Wesley, pp.425-58.

Greenberger, D. M., Horne, M. and Zeilinger, A. [1989]: 'Going Beyond Bell's Theorem', in M. Kafatos (*ed.*) *Bell's Theorem, Quantum Theory and Conceptions of the Universe*, Dordrecht: Kluwer Academic, pp. 69-72.

Griffiths, R. [1984]: 'Consistent Histories and the Interpretation of Quantum Mechanics', *Journal of Statistical Physics*, **36**, pp. 219-72.

Hartle, J. [1968]: 'Quantum Mechanics of Individual Systems', *American Journal of Physics*, **36**, pp. 704-12.

Lockwood, M. [1996a]: 'Many Minds Interpretations of Quantum Mechanics', *British Journal for the Philosophy of Science*, **47**, pp.159-88

Lockwood, M. [1996b]: 'Many Minds Interpretations of Quantum Mechanics: Replies to Replies', *British Journal for the Philosophy of Science*, **47**, pp.445-61

Loewer, B. [1996]: 'Comment on Lockwood', *British Journal for the Philosophy of Science*, **47**, pp. 229-32.

Maudlin, T. [1994]: *Quantum Nonlocality and Relativity*, Oxford: Blackwell.

Mermin, D. [1990]: 'Simple Unified Form of the Major No-Hidden Variables Theorems', *Physical Review Letters*, **65**, pp. 3373-6.

Papineau, D. [1996]: 'Many Minds Are no Worse Than One', *British Journal for the Philosophy of Science*, **47**, pp. 233-41.

Pitowsky, I. [1994]: 'George Boole's 'Conditions of Possible Experience' and the Quantum Puzzle', *British Journal for the Philosophy of Science*, **45**, pp. 95-125.

Redhead, M. [1987]: *Incompleteness, Nonlocality and Realism*, Oxford: Clarendon Press.

Saunders, S. [1998]: 'Time, Quantum Mechanics and Probability', *Synthese*, **114**, pp. 373-404.

Schrödinger, E. [1935]: 'Die Gegenvartige Situation in der Quantenmechanik', in A. Dick, G. Kerber and W. Kerber (*eds.*), *Erwin Schrödinger's Collected Papers* (Vol. 3), Vienna: Austrian Academy of Sciences, pp. 484-501.

Vaidman, L. [1998]: 'On Schizophrenic Experiences of the Neutron or Why We Should Believe in the Many-Worlds Interpretation of Quantum Mechanics', *International Studies in the Philosophy of Science*, **12**, pp.245-61.

Zeh, H. [1973]: 'Toward a Quantum Theory of Observation', *Foundations of Physics*, **3**, pp. 109-16.

Zurek, W. [1993]: 'Preferred States, Predictability, Classicality, and the Environment-Induced Decoherence', *Progress in Theoretical Physics*, **89**, pp. 281-312.